MOMTCHIL POJARLIEV AND WOLFGANG POLASEK

APPLYING MULTIVARIATE TIME SERIES FORECASTS FOR ACTIVE PORTFOLIO MANAGEMENT

Momtchil Pojarliev, INVESCO Asset Management,
Bleichstrasse 60–62, D - 60313 Frankfurt,
momtchil_pojarliev@fra.invesco.com
Wolfgang Polasek, University of Basel,
Bernoullistrasse 28, CH - 4056 Basel, Wolfgang.Polasek@unibas.ch
The authors thank the two anonymus referees
for insightful remarks which have improved the paper.
All remaining errors are ours.

1. Introduction

MARKOWITZ (1952) mean-variance efficiency is the basis of modern finance theory for asset allocation. The MARKOWITZ efficient frontier represents all efficient portfolios in the sense that all other portfolios have lower expected returns for a given level of risk measured by standard deviation. Using a classical mean-variance framework we investigate how optimal holdings for the three global regions (North America, Europe and the Pacific) depart from the benchmark (MSCI World index) weights. We estimate the optimal weights for the MSCI North America, MSCI Europe and MSCI Pacific indices for four different strategies. Portfolio one is constructed using the moving average of the past variance as a forecast for the next period. Portfolio two is based on the forecasted variance matrix from a multivariate ARCH model, specifically the BEKK model of ENGLE and KRONER (1995). For the estimation of the optimal weights of portfolio three we will use the forecasts of the first two moments of the returns of the three MSCI indices. In portfolio four we develop a simple market timing rule based on multivariate volatility forecasts.

The paper is organized as follows: In the next section, we describe the 3 MSCI indices which will be included in the portfolio analysis. In section 3 we estimate a multivariate BEKK model for the returns and the volatilities of the three MSCI indices. Section four describes the four portfolios we use in the comparison and the classical mean-variance framework for constructing the portfolios. In section five we evaluate the portfolios with the benchmark using different criteria. In the last section we summarize our results.

2. Data Description

We investigate the daily returns of the MSCI Pacific, MSCI Europe and MSCI North America indices. The time horizon is from 1 May, 1995 to 3 April, 2000 (about five years), and is divided into two parts: an in-sample-period of 800 trading days (approximately 3 years) until 23 May, 1998, which is used as the “training sample” for model estimation; and an out-of-

Table 1: Summary Statistics of the Daily Returns of the MSCI Indices from May 1st 1995 until April 3rd 2000

	Pacific	Europe	North America	World
Min	–0.0505	–0.0404	–0.0688	–0.0452
25% Quartile	–0.0070	–0.0043	–0.0041	–0.0032
Mean	0.0000	0.0006	0.0008	0.0006
Median	0.0000	0.0010	0.0007	0.0009
75% Quartile	0.0065	0.0055	0.0064	0.0046
Max	0.1082	0.0415	0.0482	0.0323
Std.Dev.	0.0131	0.0091	0.0102	0.0076
Skewness	0.60	0.24	0.45	0.40
Kurtosis	5.25	1.97	4.79	3.00

Table 2: Correlation Matrix

	Pacific	Europe	North America	World
Pacific	1.00	0.34	0.05	0.47
Europe	0.34	1.00	0.47	0.73
North America	0.05	0.35	1.00	0.83
World	0.47	0.73	0.83	1.00

sample period of 486 days, which is used for portfolio evaluation. Table 1 shows the descriptive statistics for the whole time horizon. Note that all returns are positively skewed and that the kurtosis is largest (or peaked) for the Pacific area (5.25) and smallest (or flat) for Europe (1.97). The risk measured by the standard deviation is largest for the MSCI Pacific index (0.0131) and smallest for the MSCI World index (0.0076). The lower risk for the MSCI World index can be seen as a consequence of the portfolio effect which underlies the MSCI World index.

Table 2 shows that North America dominates the World index since it has the highest correlation coefficient (0.83) followed by Europe. This is also due to the fact that the North America index has weights larger than 50% in the MSCI World index.

3. Volatility Forecasts

This section describes how multivariate time series models can improve the variance forecasts. We will compare two volatility models: the first model is the naive or historical variance (HV) model, where the moving average of past variances is used for forecasts and the second model is the multivariate BEKK model of ENGLE and KRONER (1995).

3.1 The Naive or Historical Variance (HV) Model

We use a moving (or rolling) sample of 800 observations to estimate the variance of the returns vectors $r_t = (r^P_t\ , r^E_t\ , r^A_t)' = (r_{1,t}\ r_{2,t}\ \ r_{3,t})'$. Thus, the forecasts of the variance matrix for the period t + 1 is just the variance matrix of the past 800 trading days. The fore-

Table 3: The Forecasting Performance of the HV Model

	α (t-st.)	β (t-st.)	R^2
Pacific	0.00 (–1.52)	4.72 (3.89)	0.03
Europe	0.00 (–2.42)	7.29 (5.26)	0.05
North America	0.00 (0.30)	2.65 (2.11)	0.01

casting performance is evaluated by the auxiliary regression method of PAGAN and SCHWERT (1990). Assuming zero mean returns for all time points (see Table 1) we regress the "realized volatility" (i.e. the squared returns) on a constant and the forecasted volatility.
This auxiliary regression model has the form

$$r_{i,t}^2 = \alpha + \beta\hat{\sigma}_{i,t}^2 + \varepsilon_t, \ t = 1,\dots,T; \quad i = 1,2,3. \tag{1}$$

where the $r^2_{i,t}$ are the squared returns of the i-th MSCI index and $\hat{\sigma}_{i,t}^2$ are the variance forecasts. In the case of the multivariate model they are the diagonal elements of the moving sample. The intercept α in (1) should be close to 0 and the slope about 1. The t-statistics of the coefficients for α = 0 and β = 0 are a measure for the bias in the auxiliary regression and R^2 is a measure for the overall forecasting performance. Table 3 summarizes the results of the auxiliary regressions. The forecasts performance is disappointing in terms of the R^2 measure. North America shows the smallest R^2 and the least "bias". Given the bad evaluation results for variance forecasts it would be surprising if any portfolio based on those forecasts outperforms the benchmark, i.e. the MSCI World index.

3.2 The BEKK Model

The BEKK(p,q) model, defined by ENGLE and KRONER (1995) is a special version of a vector autoregressive conditional heteroskedasticity (VARCH) model. Let r_t be the N dimensional vector of returns at time t, then we assume a multivar iate normal distribution

$$r_t \sim N(\mu, H_t) \tag{2}$$

where μ is a constant mean vector and is H_t parameterised by the following conditional covariance matrix structure:

$$H_t = A_0A_0' + \sum_{i=1}^{p} A_i\left(\varepsilon_{t-i}\varepsilon_{t-i}'\right)A_i' + \sum_{i=1}^{q} B_iH_{t-i}B_i'. \tag{3}$$

The presence of the paired transposed matrices implies a less general parameterization than the VARCH model (where vechH_t is parameterized), but allows simpler calculation of the ML estimates. Because the coefficient matrices A_0, A_i and B_i appear in pairs they guarantee the non-negative-definiteness of the conditional covariance matrix H_t. Using the AIC criterion, we select a BEKK(2,1) model for the returns of the three MSCI indices. Table 4 shows the AIC and BIC for different orders of models.
Using a moving sample of 800 trading days we forecast the volatility of the returns of the MSCI North America, MSCI Europe and MSCI Pacific indices for the out-of-the-sample period, from 25 May 1998 until 3 April 2000 (486 trading days).

Table 4: AIC and BIC for Different Orders of the BEKK Model

	AIC	BIC
BEKK(1,1)	–16388	–16135*
BEKK(2,1)	–16400*	–16035
BEKK(2,2)	–16323	–15901
BEKK(3,2)	–16164	–15658

The star (*) denotes the smallest values.

Figure 1: One Step ahead Volatility Forecasts of MSCI Europe Index
25 May 1998 until 3 April 2000

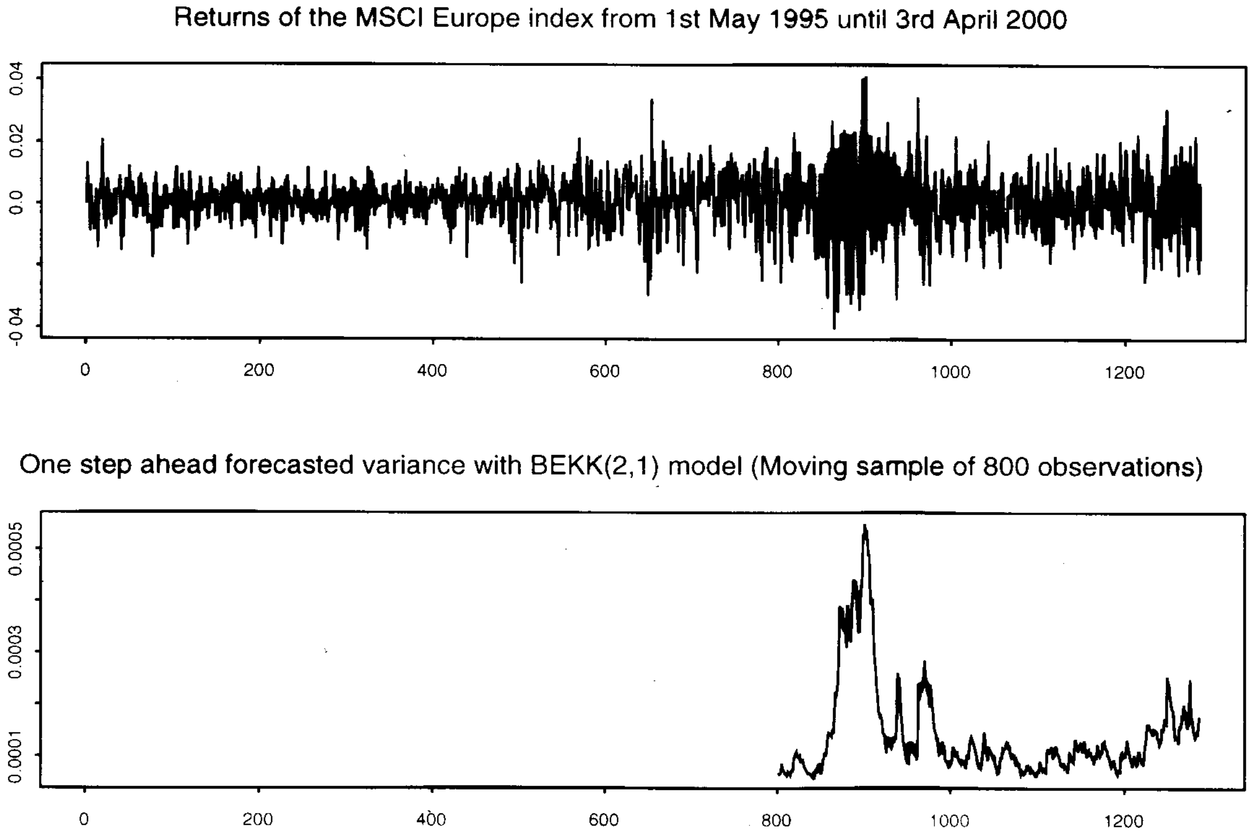


Figure 1 plots the returns and the one step ahead forecasts of the variance of the MSCI Europe index, which is taken from the second diagonal element of the forecasted variance matrix ($\hat{H}_{t+1}$). The forecasted variance matrix is obtained from (3) using the ML estimates for the coefficients of the BEKK model which was estimated by S+GARCH (1996). To evaluate the forecasting performance of the BEKK model we estimate the auxiliary regressions (1) where we use the diagonal elements of $\hat{H}_{t+1}$ as regressors, and the results are summarized in Table 5.

Comparing Tables 3 and 5 we see that the multivariate time series models perform better (according to the R^2s) than the simple naive model. In POJARLIEV and POLASEK (2000) we show that multivariate time series forecasts also improve the estimation of the value at risk of a portfolio. The next section shows that better variance forecasts improve the performance of a portfolio based on those forecasts.

Table 5: The Forecasting Performance of the BEKK Model

	α (t-st.)	β (t-st.)	R^2
Pacific	0.00 (1.50)	0.66 (3.60)	0.03
Europe	0.00 (1.20)	0.81 (7.76)	0.11
North America	0.00 (1.02)	0.87 (4.16)	0.03

4. Portfolio Construction

Investors prefer portfolios with larger mean returns and lower risk (measured in standard deviation) and they will accept more risk only if they get higher returns as compensation. This means that all investors should hold portfolios on the mean-variance frontier (see GRINOLD and KAHN, 1995). Any portfolio return on the frontier can be constructed as a combination between the market portfolio and the risk-free interest rate. The proportion of these two funds depends on the utility functions of investors. If an investor is extremely risk-averse she or he will invest only in the money market to get the risk-free rate.

Let w be the weights vector of the N assets, μ the vector of the expected returns of the N assets and H the variance matrix of the returns, then the portfolio variance is given by $\sigma^2_p = w'Hw$ and the portfolio return $\mu_p = w'\mu$. The optimization problem of a mean-variance portfolio in the absence of a risk-free asset is (see e.g. CAMPBELL et al. 1997):

$\min w'Hw$

subject to

$w'\mu = \mu_p$ and $w'\iota = 1$,

where ι is a vector of ones. We call the solution of this optimization problem the μ-fixed minimum variance (μMV) portfolio since μ_p is fixed like a target:

$$w_p = g + h\mu_p. \tag{4}$$

The target μ_p is a pre-specified portfolio return and g and h are (N x 1) vectors,

$$g = \frac{1}{D}\left[B\left(H^{-1}\iota\right) - A\left(H^{-1}\mu\right)\right] ,$$

$$h = \frac{1}{D}\left[C\left(H^{-1}\mu\right) - A\left(H^{-1}\iota\right)\right]$$

and $A = \iota' H^{-1}\mu$, $B = \mu' H^{-1}\mu$, $C = \iota' H^{-1}\iota$, and $D = BC - A^2$. A portfolio without a target is called the *global minimum-variance* portfolio if the weights vector is computed by the simpler formula

$$w_{GMV} = \frac{1}{C} H^{-1}\iota \tag{5}$$

4.1 The Historical Variance (HV) Portfolio

Using the moving average forecasts of the variance matrix in section 3.1 we derive the weights for the HV portfolio from the GMV formula in equation (5). Since the portfolio weights depend only on the precision matrix H^{-1} (or inverse variance matrix).

4.2 The Global Minimum-Variance (GMV) Portfolio

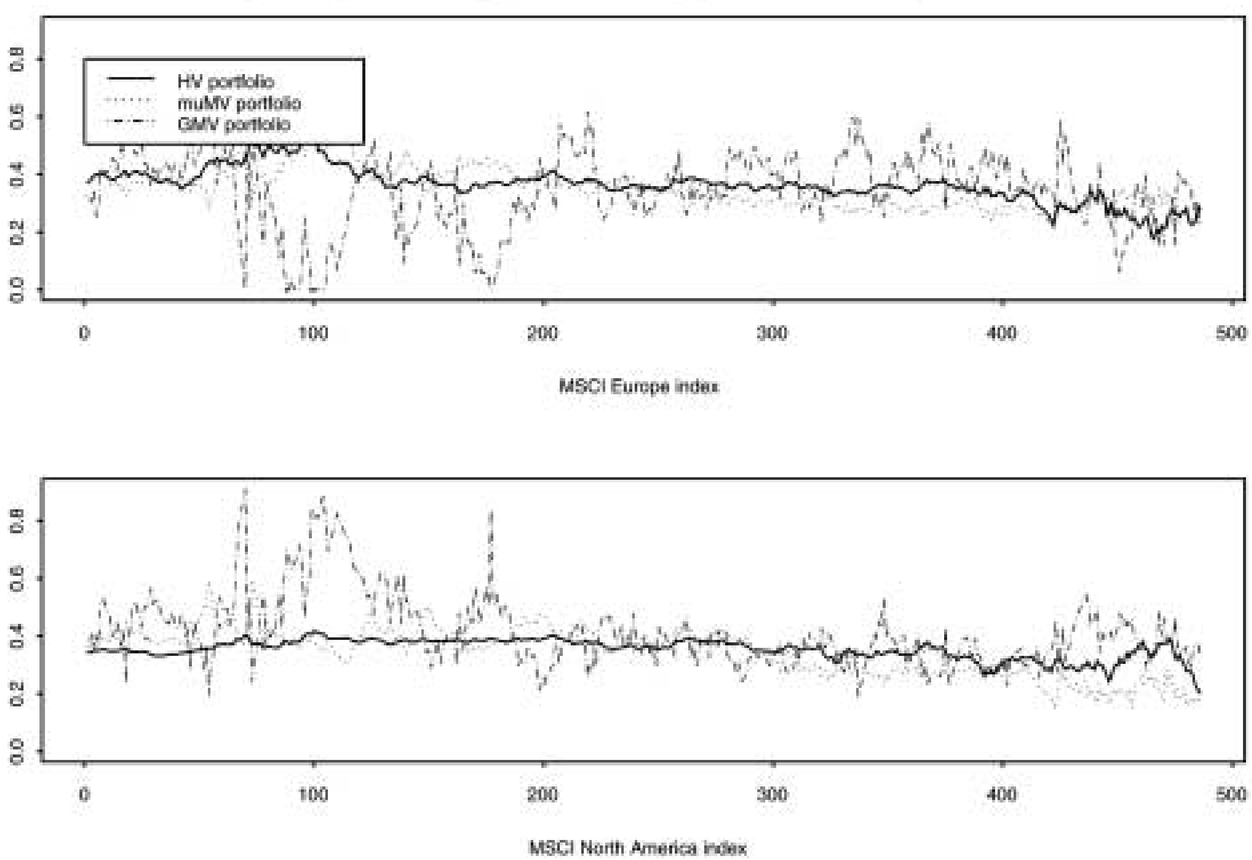
The one step ahead forecast of the variance matrix of the returns vectors of the MSCI Pacific, MSCI Europe and MSCI North America indices with the BEKK (2,1) model from section 3.2 are used instead of the simple moving average to estimate the weights vector of global minimum-variance (GMV) portfolio for the out-of-sample period. To avoid short positions we apply the restriction that all portfolio weights are non-negative ($w \geq 0$).

4.3 The µ-fixed Minimum Variance (µMV) Portfolio

We investigate if additional forecast information can improve portfolio performance. Thus, instead of only using the precision matrix we apply the forecasted returns in the portfolio decision problem.

The weights of portfolio 3 (μMV portfolio) are computed by equation (4). The required inputs

Figure 2: Comparison of optimal Portfolio Weights for Europe (Upper Panel) and North America (Lower Panel): HV, μMV and GMV Portfolio
25 May 1998 until 3rd April 2000



are: the variance matrix H, the vector of the expected asset returns μ and the targeted portfolio return μ_p. We forecast the variance matrix and the returns vectors by the BEKK(2,1) model. As portfolio return μ_p we choose 12.5% per year or 0.05% per trading day.

Figure 2 shows the weights of the three portfolios for the out-of-sample period for the MSCI Europe index and the MSCI North America index. The weights for the Pacific index are suppressed since the weights sum up to 100%. We can see that the additional forecast of the return vector μ_{t+1} implies more stable weights for the μMV portfolio than for the GMV portfolio. The use of the historical average as a forecast for the variance matrix of the HV portfolio results in weights that are more stable over time than the other portfolio weights.

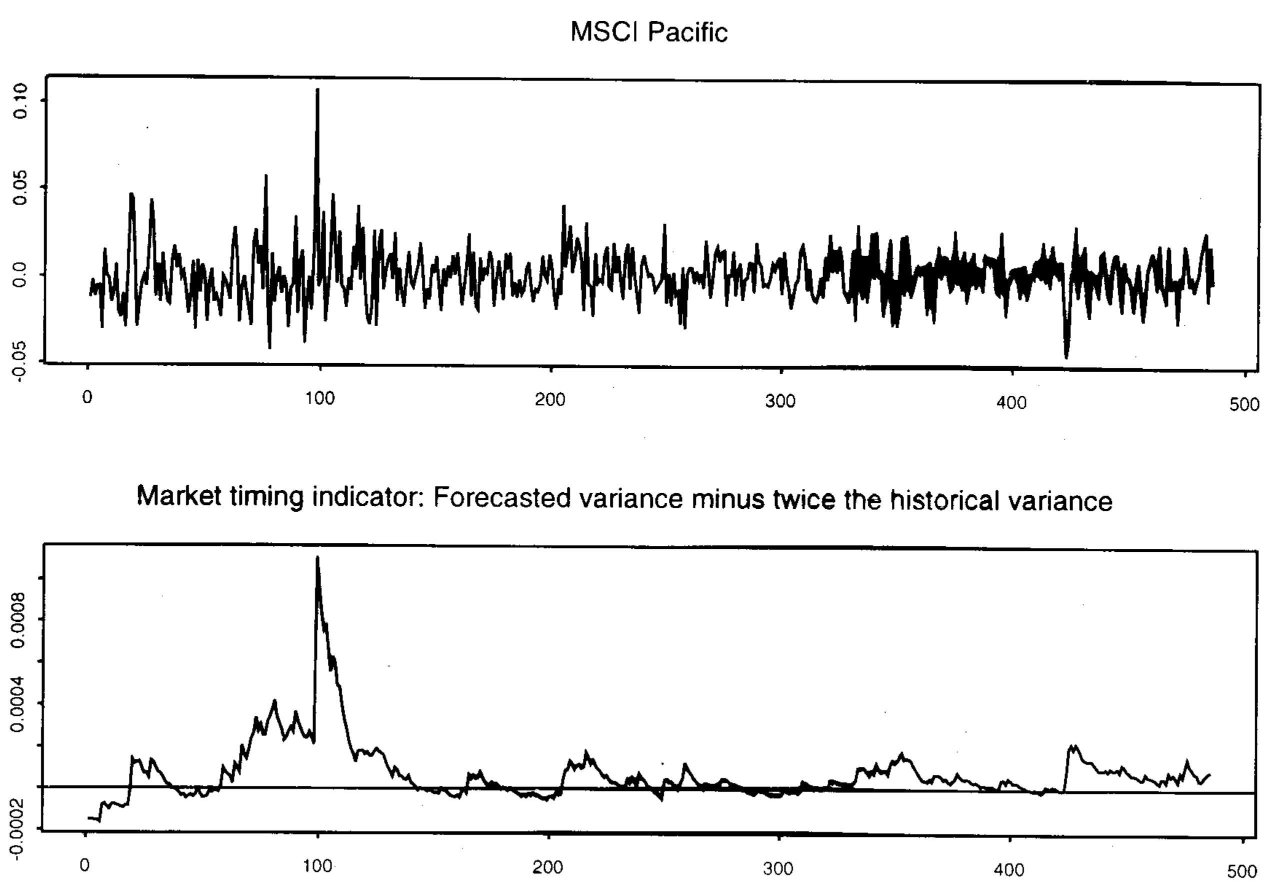
4.4 The Market Timing Portfolio

The market timing strategy is based on expected excess return and a switching rule between assets. The portfolio managers invest either in the market or in the risk-free asset: According to forecasts for the excess returns of the benchmark, they invest in the market if the excess returns are positive or they switch into cash if the excess returns are negative. COCHRANE (1999) shows that a market timing strategy (based on a regression model of returns on dividend price ratios) yields a portfolio return with an increased Sharpe ratio. He shows that a market timing strategy can almost double the average returns of the portfolio over a five year period. In the next subsection we develop a simple market timing rule based on volatility forecasts.

Market Timing Rule based on Volatility Forecasts

Assets with higher volatility have higher returns because of the potential trade-off between risk and return. Thus, we propose an aggressive market timing rule: If the forecasted volatility is twice as high as the historical volatility, we invest in the market, otherwise we invest in the risk-free asset. Note that switching into the risk-free asset will decrease the portfolio variance. The lower panel of Figure 3 shows the market timing indicator for the Pacific region: We invest in the Pacific index if the forecasted (Pacific) variance is twice the historical variance. The horizontal line marks the days in which the doubled forecasted (Pacific) variance is equal to the historical variance (the market timing indicator is equal to zero). Thus, we invest in the Pacific index, when the indicator is over the horizontal line and visa versa. Similarly, we have defined the market timing indicator for the European and the North American region. The portfolio return for each time period is now a potential mixture of the risk-free rate and the regional index returns. Note that this strategy does not imply that we will always get returns above the

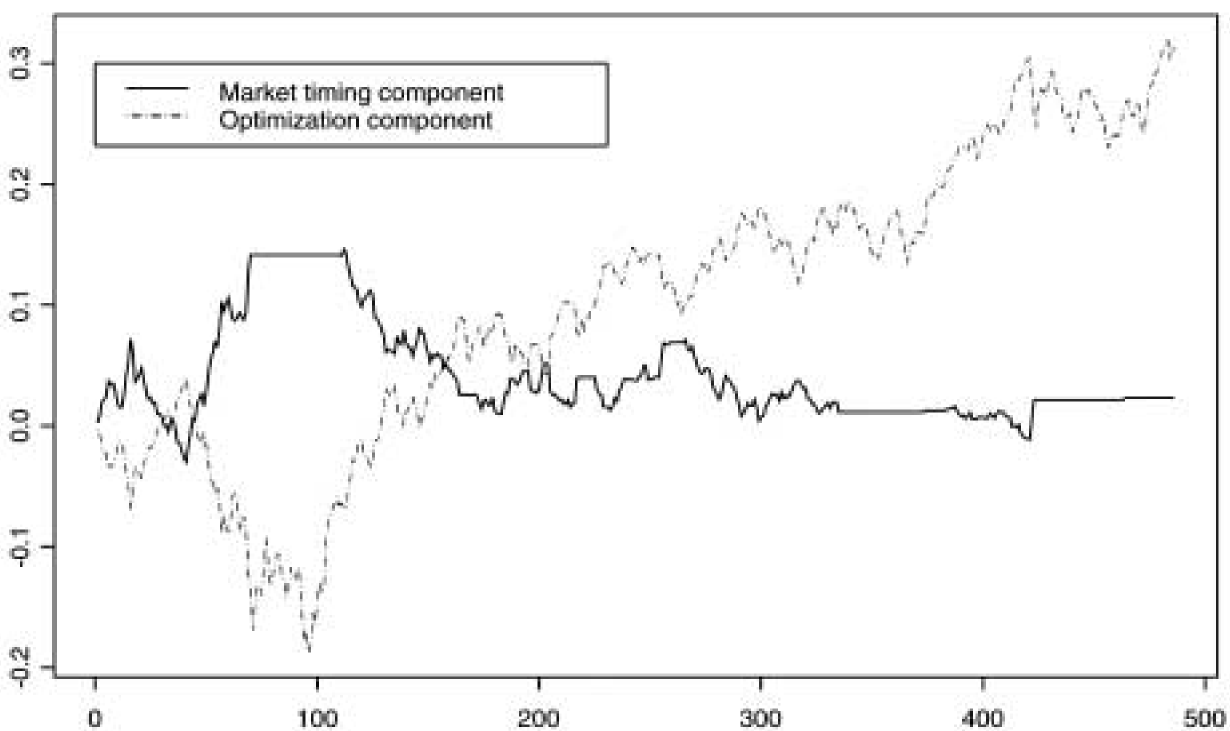
Figure 3: Returns and the Market Timing Indicator



risk-free rate: If high volatility periods produce returns under the risk-free rate, we might end up with an overall portfolio return (in one or more periods) below the risk-free rate. If the indicator is positive, we invest in the appropriate MSCI index, otherwise we invest in cash. In the multivariate case, we invest in the index with the weights computed as for the GMV portfolio. For the risk-free rate we assume 5% p.a. (as an alternative we could choose to invest in bonds).

Figure 4 illustrates the overall gain of the market timing strategy. The cumulative portfolio returns are decomposed into two components: 1) the baseline portfolio gain which stems from the returns of the GMV portfolio and 2) the additional gain based on the overall market timing rule. This MT component is positive except a short period in summer 1998. This shows that we could utilize the trade-off between risk and returns in the regional portfolio. The plateau's in the MT component mark those periods where the GMV portfolio – which is seen as dotted line – determines the return of the MT portfolio. The non-plateau period are all those days where at least one region did not invest in the regional index.

Figure 4: The Cumulative Components of the Market Timing Portfolio



Note: The dotted line shows the GMV returns without MT. The bold line shows the additional yield using the MT volatility strategy. The plateau's are the periods with high volatility in all 3 regions: then the GMV and the MT portfolio give the same returns

5. Comparison

We compare the portfolio performances with the MSCI World index for the out-of-sample period (25 May 1998 – 3 April 2000) using the following criteria:

- mean return per year (in percent)
- standard deviation per year (in percent)
- cumulative return for 2 years and year to date (in percent)
- SHARPE ratio
- success rate

The SHARPE ratio S_p is defined as the expected excess return of portfolio P divided by the risk of portfolio P

$$S_p = \frac{r_p - r_{riskfree}}{\sigma_p}, \tag{6}$$

where the risk-free rate $r_{riskfree}$ is assumed to be 5% per year. The correlation between the given portfolio and the market portfolio is the ratio of Sharpe measures (see HARVEY and ZHOU, 1990). We define the success rate as the percentage of times (in months) in which the portfolio returns beat the benchmark returns.

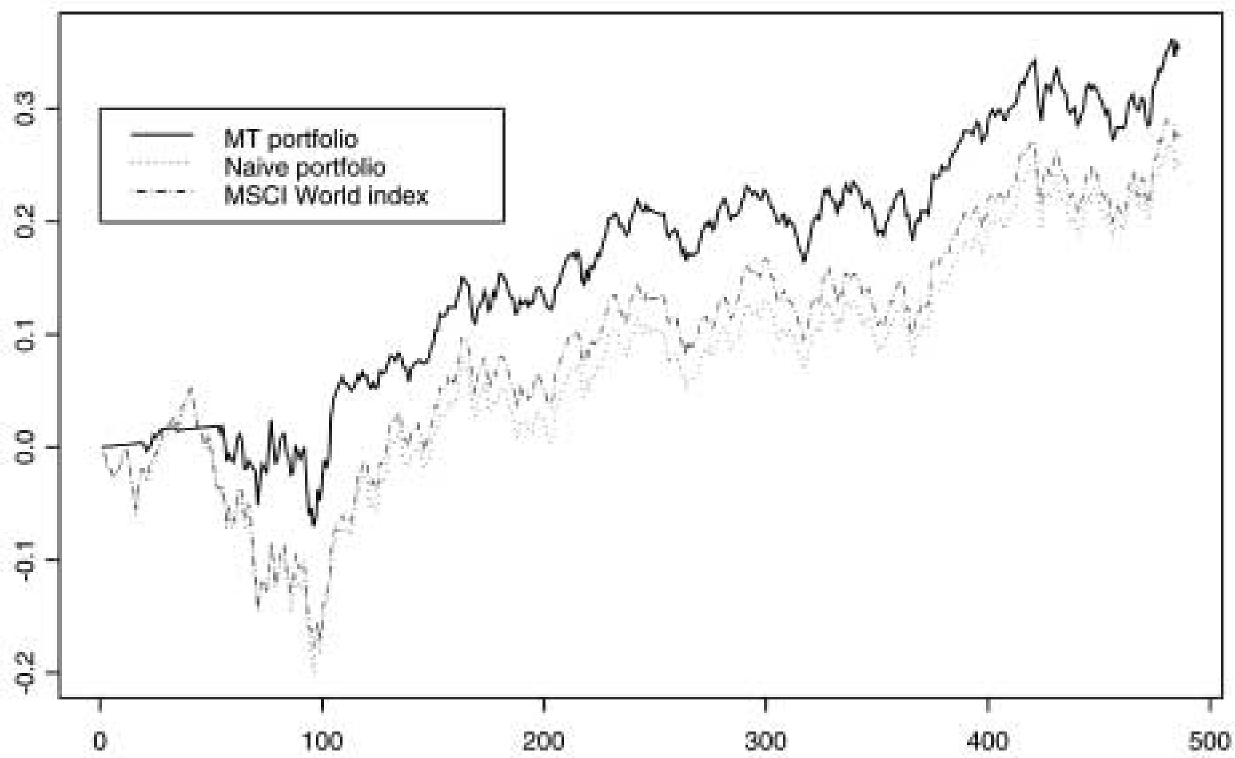
Table 6 summarizes the evaluation of the portfolios by the five criteria. The HV portfolio performs worse than the benchmark, while the GMV portfolio, based on the one step ahead forecast of the variance matrix in the BEKK(2,1) model dominates the benchmark (MSCI World index): it exhibits larger mean returns and smaller standard deviation. The returns of the µMV portfolio are better than the benchmark, but the gains are smaller than in the GMV portfolio. The µMV portfolio performance is the worse in the year-to-date comparison (the year-to-date cumulative returns start on 1st January 2000), but has much more stable portfolio weights, which eventually leads to smaller transaction costs. Portfolio four (the MT portfolio) based on the market timing strategy dominates all other portfolios.

Figure 5 compares the cumulative returns of the naive or HV portfolio and the MT portfolio with the benchmark. Note that the MT portfolio beats the benchmark by almost 4% points on a annual basis and is also the best in the year-to-date comparison. The Sharpe ratio is also the largest for the MT portfolio and this is also due to a smaller standard deviation of the portfolio return. The success rate is low for all portfolios and surprisingly less than 50% for the winner, the markt timing portfolio. This might be a sign that the higher yields from the MT strategy might be concentrated in a few periods.

Table 6: Performance of the Portfolios and the Benchmark (MSCI World Index)
Cumulative returns for 2 years (from 25 May 1998 until 3 April 2000) and year-to-date returns for the year 2000

	HV portfolio	GMV portfolio	µMV portfolio	MT portfolio	benchmark
mean returns	12.84%	16.39%	14.66%	18.17%	14.13%
st. dev.	14.90%	14.80%	14.99%	12.72%	15.14%
cum. returns					
2 years	24.96%	31.86%	28.5%	35.31%	27.46%
year to date	–0.06%	0.90%	–0.23%	1.37%	0.42%
SHARPE ratio	0.52	0.76	0.64	1.05	0.60
success rate	51.00%	51.80%	50.62%	49.59%	

Figure 5: Cumulative Returns from 25 May 1998 until 3 April 2000 for the HV and the MT Portfolio and the Benchmark (MSCI World Index)



6. Conclusion

This paper describes a multivariate time series model that is used to improve volatility forecasts of stock indices. We show how portfolio performance for a 3-dimensional portfolio model can be improved by modelling the returns of the MSCI indices of North America, Europe and the Pacific region. The MSCI World index is used as a benchmark. Furthermore, we have proposed a new market timing portfolio based on volatility forecasts. The market timing strategy invests only in the stock index if the forecasted volatility is twice as large as the historical volatility. The comparison of the portfolios shows that the market timing (MT) portfolio performs very well. It has higher cumulative returns and smaller standard deviations for the last 2 years. Our approach shows that there exist trading strategies which can successfully exploit the trade-off between risk and returns. The volatility predictions gained through multivariate time series models can be successfully transformed into higher portfolio yields by quantitative portfolio strategies if the right combination of volatility modelling and portfolio strategy can be found. Many more market timing strategies are possible and it remains to be seen if and when they are successful. We conclude that research in market timing strategies might become more important in future for fund managers who want to outperform benchmarks in a more technical way.

REFERENCES

CAMPBELL, J. Y., A. W. LO and A. C. MACKINLAY (1997): The Econometrics of Financial Markets, Princeton: Princeton University Press.

COCHRANE, J. H. (1999): "Portfolio Advice for a Multifactor World ", National Bureau of Economic Research, Working Paper 7170, http://www.nber.org/papers/w7170.

ENGLE, R. F. and K. F. KRONER (1995): "Multivariate Simultaneous GARCH", Econometric Theory 11, pp. 122–150.

GRINOLD, R. C. and R. N. KAHN (1995): "Active Portfolio Management", McGraw-Hill.

HARVEY, C. R. and G. ZHOU (1990): "Bayesian inference in asset pricing tests", Journal of Financial Economics 26, pp.221–256.

MARKOWITZ H. (1952): "Portfolio Selection", Journal of Finance 7, pp. 77–91.

MATHSOFT (1996): “S+GARCH User's Manual”, MathSoft, Seattle.

POJARLIEV, M. and W. POLASEK (2000): "Value at Risk estimation for stock indices using the Basle Committee proposal from 1995", University of Basel, WWZ-Discussion Paper 00/07.

Momtchil Pojarliev and Wolfgang Polasek: Applying Multivariate Time Series Forecasts for Active Portfolio Management

Momtchil Pojarliev and Wolfgang Polasek: Applying Multivariate Time Series Forecasts for Active Portfolio Management

Momtchil Pojarliev and Wolfgang Polasek: Applying Multivariate Time Series Forecasts for Active Portfolio Management

Momtchil Pojarliev and Wolfgang Polasek: Applying Multivariate Time Series Forecasts for Active Portfolio Management

Momtchil Pojarliev and Wolfgang Polasek: Applying Multivariate Time Series Forecasts for Active Portfolio Management

Momtchil Pojarliev and Wolfgang Polasek: Applying Multivariate Time Series Forecasts for Active Portfolio Management

Momtchil Pojarliev and Wolfgang Polasek: Applying Multivariate Time Series Forecasts for Active Portfolio Management

Momtchil Pojarliev and Wolfgang Polasek: Applying Multivariate Time Series Forecasts for Active Portfolio Management

Momtchil Pojarliev and Wolfgang Polasek: Applying Multivariate Time Series Forecasts for Active Portfolio Management

Momtchil Pojarliev and Wolfgang Polasek: Applying Multivariate Time Series Forecasts for Active Portfolio Management

Momtchil Pojarliev and Wolfgang Polasek: Applying Multivariate Time Series Forecasts for Active Portfolio Management